Comp 775: Image Registration

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Establishing a geometric transformation

\[ x' = h(x) = x' = x + \Delta x \]

relating points in one image to points in another.
Purpose of Image Registration

Establishing a geometric transformation

\[ x' = h(x) = x' = x + \Delta x \]

relating points in one image to points in another.

Needed (as with segmentation)

- Regularization Energy (Geometric atypicality)
- Data Similarity Measure (geometry-to-data match)
- Optimization Scheme

And also

- Transformation Model
- Interpolation Model
Registration Example

Skulls of a human, a chimpanzee and a baboon and transformations between them

D'Arcy Thompson
Important: Make sure to get your coordinate systems straight.

• Image data is more than just a 3D-array of numbers
• Images may have been acquired in different coordinates systems
• Orientationally dependent data (e.g., tensors) may have additional coordinate systems.

To be certain the alignment can work properly work in World coordinates
Right-handed coordinate systems

Right handed coordinate system.

1st finger

2nd finger

Image: Atkinson
Coordinate Systems

L: Left
P: Posterior
S: Superior
R: Right
A: Anterior
S: Superior

Supine (face-up) patient.

Image: Atkinson
Coordinate Systems

## Coordinate Systems

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<th>Axial</th>
<th>Sagittal</th>
<th>Coronal</th>
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<tr>
<td>anterior</td>
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<td>right</td>
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<td>right</td>
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<td>left</td>
<td>posterior</td>
<td>left</td>
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</tbody>
</table>

**Important:** Make sure to get your coordinate systems straight. What is right and what is left for example?
Pixels/Voxels vs. World Coordinates

Image: ITK Registration Guide
Image Coordinates

Spacing (Sx)

Origin (Ox, Oy)

Spacing (Sy)

Image: ITK Registration Guide
Image coordinates
Coordinate System Conversions

Image Grid

Physical Coordinates

Images: ITK Registration Guide

Space Transform

Physical Coordinates
Coordinate Systems, Example

NRRD0005
# Complete NRRD file format specification at:
# http://teem.sourceforge.net/nrrd/format.html
type: unsigned short
dimension: 4
space: right-anterior-superior
sizes: 256 256 78 59
thicknesses: NaN NaN 2 NaN
space directions: (-0.9375,0,0) (0,-0.9375,0) (0,0,1.7)
none
centerings: cell cell cell ???
none
kinds: space space space list
endian: little
encoding: gzip
space units: "mm" "mm" "mm"
space origin: (-136.14,-130.248,-29.8626)
measurement frame: (1,0,0) (0,1,0) (0,0,1)
data file: caseD00959-dwi-EdCor.raw.gz
modality:=DWMRI
DWMRI_b-value:=700
DWMRIGradient_0000:= 0.000000 0.000000 0.000000
DWMRIGradient_0001:= 0.000000 0.000000 0.000000
....

Image: Kindlmann
Registering What to What?

Image Registration

Registration Types

Extrinsic

Intrinsic

Registration features derived from image itself
Registration Types

Landmark based

Segmentation based

Image based

Images: Alain Pitiot
Registration Types

Intra-subject registration
(Within subject)

Images: Alain Pitiot
Inter-subject registration (Between subject)

Images: Alain Pitiot
Registration Types

Subject/Atlas Registration

Images: Alain Pitiot
This can be used as a segmentation method or to add spatial prior information to a segmentation method = Atlas-based segmentation

Images: Alain Pitiot
Multi-Modality Registration

X-ray CT

MRI

Requires special care with respect to the similarity metric. Use for example mutual information.
Image Registration Components

Fixed Image

Moving Image

Registration Method

Metric

Optimizer

Interpolator

Transform

Image: ITK Registration Guide
Interpolation

Determining function values off the grid.

Nearest Neighbor Interpolation:
   Good for label maps (does not create any new values)

Linear Interpolation

B-Spline Interpolation
   Smooth, piecewise bi(tri)-cubic
   Achievable by successive subdivision, when
c      discrete interpolation is what is desired
   Used for interpolation (but does not exact match control pts),
      but also to represent spatial transformations (see later)
Transformations

**Low-dimensional**

- **Rigid**: translation + rotation
- **Similarity**: translation + rotation + scale
- **Affine**: translation + rotation + scale + shear

**High-dimensional**

- **Elastic**: regularization of displacements (can fold)
- **Fluid**: regularization of velocities
  (can avoid folding)
Transformations

Translation
\[ f : \mathbf{x} \mapsto \mathbf{x} + \mathbf{t} \]

Similarity Transform
\[ f : \mathbf{x} \mapsto sR\mathbf{x} \]

Affine Transform
\[ f : \mathbf{x} \mapsto A\mathbf{x} + \mathbf{t} \]
Example affine transformation

\[ x' = Ax + \Delta x \]
Example Registration

Fixed Image  Moving Image  Registered Moving Image

Images: ITK Registration Guide
Landmark-based registration
Simplest parametric ones are of course rigid, similarity, affine, ...

More parameters = more flexibility

- **localized**
  - **global** (no localization)
    - Trigonometric bases (Fourier/Sine/Cosine bases)
- **regular grid**
  - **irregular sampling points**
    - Adaptive bases, Thin-plate Splines (TPS)
- **dense**
- **sparse**
  - Free-form Deformations (FFD)

Image: Zikic
Geometry and Warps Via Landmarks

- Compute $\Delta x(x)$ or a decomposition into translations, rotations, magnifications, & ellipse forming deformations
- Energy options
  - Procrustes energy: for global alignment
  - Thin plate spline bending energy: for exact matching warp, possibly incl. alignment
- Approximate matching warps:
  - Elastic energy
  - Diffeomorphic flow, e.g., by fluid energy
  - “Freeform” (b-spline) deformations
Geometry and Warps via Landmarks: Issues

- Produces general warp?
- Limited to non-folding warps?
- Energy captures all aspects of warp?
- Symmetric re static vs. moving images?
Want to avoid implausible transformations

For example, diffeomorphic transformations: “Bijective, smooth, with a smooth inverse”

Image: Staring
Diffeomorphisms

Is this all that is needed? Are we missing something?
Diffeomorphisms

It seems like a homeomorphism is all we want. Because it
• prevents folding and
• does not allow for tearing either.

What else could we ask for?

The function

\[ f : x \mapsto x^3, \quad f^{-1} : x \mapsto x^{\frac{1}{3}}, \quad x \in \mathbb{R}, \]

is homeomorphic. But the derivative of \( f^{-1} \) is not defined at 0.

A diffeomorphism is a smooth bijective mapping with a smooth inverse.
Geometric typicality metrics: PDM: Procrustes

• Align shape before warp transformation
  – translation, e.g., to center of mass
  – scaling(?): $|\mathbf{x}| = 1$
  – rotation to minimize $|\mathbf{x} - \mathbf{x}_{\text{std}}|$

• Metric = $\sum_i |x^i - x^{\text{std}}_i|^2$
  – Has statistical variant

• $O(n)$, with $n =$number of points
• Symmetric
Thin plate spline deformation energy

- Elastic warp in each variable
- Based on landmark primitives
- Minimizing integrated 2nd derivs²
  - So smooth
- Not necessarily diffeomorphic; may produce folding
- Not symmetric
- Due to Bookstein: Ref: [Dryden & Mardia, *Statistical Shape Analysis*]
- $O(n^2)$
Splines w/ Landmarks

Image: Younes
Thin-plate Splines

Images: University of Vienna

Warping a human skull into a chimpanzee skull.
Thin plate spline deformation energy

Method of landmark matching based on finding the mapping \( fcn \).

\[
f(x, y) = a_0 + a_x x + a_y y + \sum_{i=1}^{n} w_i U(|p_i - (x, y)^T|)
\]

where \( U(r) = -r^2 \log(r^2) \)

that matches landmark points to each other while minimizing

\[
l_f = \int \int \left( \left( \frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 \right) \, dx \, dy
\]

This mapping is not guaranteed to be diffeomorphic.
B-Spline Transformations “Freeform Deformation” [Rueckert]

- **Grid** of control points
- Value at each control point
  - Can be scalar in general
  - For registration, a displacement vector
    - 3 scalars, each separately interpolated
- Patchwise bi(tri)-cubic; smooth at patch boundaries
- Raw form can fold; there is diffeomorphic variant
  - Successive small freeform changes
Flowing images into each other. Mapping function \( h(x) = \phi(x, 1) \) given through the ODE

\[
\frac{d\phi(x, t)}{dt} = v(\phi(x, t)), \quad t \in [0, 1], \quad \phi(x, 0) = x.
\]

Minimize smoothness cost subj. to landmark constraints \( (h(x_n) = y_n) \)

\[
\hat{v}(\cdot) = \arg\min_v \int_0^1 \int_{\Omega} \|Lv(x, t)\|^2 \, dx \, dt.
\]

This is guaranteed to give a diffeomorphic \( h \) for suitable \( L \) (for example \( L = I(-\nabla^2 + c) \) works).
Diffeomorphic Landmark Matching

Image from Joshi.

Left: target image, middle: diff. landmark matching, right: small displacement matching.
Image-based registration
Elastic-type versus fluid-type registration

**Elastic Registration**

Regularization on the displacement field \( (u) \)

\[
u = \underset{u}{\operatorname{argmin}} \int \Psi_s^u [u] + \frac{1}{\sigma^2} \Psi_d [u, I_0, I_1] \, d\Omega\]

**Fluid Registration**

Regularization on the time dependent velocity field \( (v) \)

\[
v = \underset{v}{\operatorname{argmin}} \iint \Psi_s^v [v] \, d\Omega \, dt + \frac{1}{\sigma^2} \int \Psi_d [u, I_0, I_1] \, d\Omega\]
Elastic Transformation

Images: Ashburner
Deformable registration (dense deformation fields)

Many registration methods available.
Displacement-regularized registration

Combination of
- regularization of displacement field \( u \) and
- image similarity measure (SSD, correlation, MI, etc.)

\[
\begin{align*}
\mathbf{u}^* &= \arg\min_u S[u] + \frac{1}{\sigma^2} D[u, I_0, I_1]
\end{align*}
\]

Multiple options for regularization also
- diffusive
- curvature
- elastic
- ...

Image Registration  Registration Models
Diffusion Regularization (=Optical Flow)

Enforces smoothness of the displacement fields (component by component)

\[ S[u] = \frac{1}{2} \sum_{l=1}^{d} \int_{\Omega} \| \nabla u_l \|^2 \, dx = \frac{1}{2} \int_{\Omega} \| \nabla u_1 \|^2 + \| \nabla u_2 \|^2 \, dx \]

Simplest model, as used for example in optical flow (Gradients will result in Laplacian terms -> smoothing)
Optical Flow

\[ E(u^x, u^y) = \int_{\Omega} \left( I_t + I_x v^x + I_y v^y \right)^2 + \alpha \left( \| \nabla v^x \|^2 + \| \nabla v^y \|^2 \right) \, d\Omega \]

Images: Slides of Bill Freeman
Curvature Regularization

Regularization based on the Laplacian of the displacements

\[ S[u] = \frac{1}{2} \sum_{l=1}^{d} \int_{\Omega} (\Delta u_l)^2 \, dx \overset{2D}{=} \frac{1}{2} \int_{\Omega} (\Delta u_1)^2 + (\Delta u_2)^2 \, dx \]

Invariant to affine transformations (due to second derivatives).
Elastic Regularization

Based on physical model of linear elasticity

\[ S[u] = \int_{\Omega} \frac{\mu}{4} \sum_{j,k=1}^{d} (\partial_{x_j} u_k + \partial_{x_k} u_j)^2 + \frac{\lambda}{2} (\text{div } u)^2 \]

- \( \text{rigidity} \)
- \( \text{change in material volume} \)
- \( \text{elastic potential} \)

\( \mu, \lambda \): Lame constants (control elastic behavior)
Elastic Regularization

Elasticity model is the continuum mechanics equivalent to the spring model

\[ E[u] = \frac{1}{2} ku^2 \]

Can compute the force by differentiation

\[ F[u] = -\frac{dE(u)}{du} = -ku \]

In the continuum, the force is obtained through the variation

\[ f = \mu \Delta u + (\lambda + \mu) \nabla \text{div}(u) \]

which, needs to be balanced w/ force form similarity measure
Fluid flow registration

Fluid flow setup [Miller et al.]:

\[
E(v) = \int_0^1 \|v\|_V^2 \, dt + \frac{1}{\sigma^2} \|I_0 \circ \Phi_{1,0} - I_1\|_{L_2}^2
\]

What is the best velocity field, \(v\), to deform one image into the other?
Fluid flow registration

Fluid flow setup [Miller et al.]:

\[E(v) = \int_0^1 \left\| v \right\|_V^2 \, dt + \frac{1}{\sigma^2} \left\| I_0 \circ \Phi_{1,0} - I_1 \right\|_{L_2}^2\]

Complex optimization problem:
- \(v\) depends on time and space
- requires solution of full space-time problem
- often approximately solved using a greedy algorithm (greedy versions of LDDMM, Demons, ...)

Test cases

Test cases
Non-parametric: Elastic- vs. fluid-type registration

Images: Christensen
Standard test cases to assess registration behaviour for different registration algorithms

Images: Modersitzki
Regularization of displacements hinders large deformations

Images: Modersitzki
Elastic Registration

Images: Modersitzki
Elastic Registration

As for diffusive regularization, the regularization of displacements discourages large displacements

Images: Modersitzki
Fluid Registration [Christensen, Joshi, Miller]

Since fluid registrations regularize velocities they allow for large deformations.

Images: Modersitzki
General Issues
Warp Transformation on Landmarks or Image Data (or both)

- Typically pre-align
  - Result is different if you don’t pre-align

- Objective function terms:
  - Regularization energy
    - Weight value is hard to determine, esp. since two terms are in different units
  - Geometry-to-data match
    - For landmarks
    - For image features

- Does not give exact match due to regularization
- Often will still fall into local energy minima
  - Especially if you do not pre-align
Multi-Resolution

Fixed Image

Registration

Registration

Registration

Moving Image

Speeds-up convergence
Helps w/ local minima

Images: ITK Registration Guide
There may be more than one (visually identical) solution, depending on the transformation chosen.

Images: Modersitzki and Haber
Registration Quality $\not\approx$ Visual Quality

Images: Modersitzki and Haber
Registration Quality ≠ Visual Quality

Images: Modersitzki and Haber
Registration Quality $\not\approx$ Visual Quality

With constraint of bone rigidity.

Without constraint.

Images: Modersitzki and Haber
Registration w/ Point Correspondences

• Landmarks only
  – Sum of
    • Atypicality energy term prevents perfect correspondence
    • Landmark match: typically sum of squared distances
    • No perfect landmark match is achieved
  – Or curved diffeomorphic paths plus piecewise TPS interpolation

• Image registration with point correspondence
  – Image match + landmark match + geom energy
    • No perfect landmark match is achieved
  – Or landmark-constrained registration
Optimization

- By gradient descent on objective function $J(T)$ over space of possible functions $T$
- So what is the gradient $\nabla_T J$ of a scalar function of a function of typically many variables?
  - Calculus of variations
- With $\nabla_T F$, gradient descent is $\partial I/\partial t = \nabla_T J$
  - Euler-Lagrange optimization

Gradient descent is simplest method. May not have fastest convergence. Other optimization methods exist.
Some Background Material

Registration tutorial
- Some of the slides only
- Many details on underlying theory

http://campar.in.tum.de/DefRegTutorial/WebHome
Tools supporting parametric registration

- B-splines
  - Insight Toolkit [itk.org](http://www.itk.org)
  - Elastix [elastix.isi.uu.nl](http://elastix.isi.uu.nl)
  - FAIR
- Image Registration Toolkit (IRTK)
  - [http://www.doc.ic.ac.uk/~dr/software/index.html](http://www.doc.ic.ac.uk/~dr/software/index.html)

- Thin-plate splines
  - Elastix
Tools supporting non-parametric registration

- Diffusive Registration
  - FAIR
  - Free-surfacer
- Curvature Registration
  - FAIR
- Membrane/Bending Energy
  - FSL [http://www.fmrib.ox.ac.uk/fsl/](http://www.fmrib.ox.ac.uk/fsl/)
- Demons
  - ITK (Insight Journal)
- Diffeomorphic Demons
  - ITK (Insight Journal)
- Fluid flow
  - FAIR
Tools supporting non-parametric registration

- General diffeomorphic fluid flow
  - ANTs (Advanced Normalization Tools)
    www.picsl.upenn.edu/ANTS/lastix.isi.uu.nl
  - FRAT (Fluid Registration and Atlas Toolkit)
    www.nitrc.org/projects/frat
  - SPM (recommends DARTEL)
    http://www.fil.ion.ucl.ac.uk/spm/
  - AtlasWerks (has GPU implementations)
    http://www.sci.utah.edu/software.html
3D Slicer: slicer.org

- Integrates a large selection of registration algorithms
- Parametric and non-parametric


Images: slicer.org
# 3D Slicer registration capabilities

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<tr>
<th>Module</th>
<th>mode</th>
<th>type / DOF of motion</th>
<th>speed</th>
<th>data-type</th>
<th>options</th>
<th>in- &amp; output</th>
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<td>Expert Pipeline Registration</td>
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Large collection of registration algorithms

Images: slicer.org
Questions

Questions?