Shape Descriptions w/ Fourier Series

Start with parametric description of the curve. (What parametrization, i.e., what sampling?) Then represent the coordinate functions $x(p)$ and $y(p)$ as a Fourier series.
Functions have global support.
Shape Description w/ Fourier Series

The most basic shape: the circle

\[ x(p) = \cos(2\pi p), \quad y(p) = \sin(2\pi p) \]
Shape Description w/ Fourier Series

\[ x(p) = 3 \cos(2\pi p), \quad y(p) = \sin(2\pi p) \]

An ellipse by stretching one direction
Shape Description w/ Fourier Series

\[ x(p) = \cos(2\pi p), \quad y(p) = \sin(2\pi p) + \cos(4\pi p) \]

A “bent” ellipse by adding a higher order term
Shape Description w/ Fourier Series

\[ x(p) = 3 \cos(2\pi p), \quad y(p) = \cos(2\pi p) + \sin(2\pi p) \]

A tilted ellipse
Shape Description w/ Fourier Series

A bent and tilted ellipse

\[ x(p) = 3\cos(2\pi p), \quad y(p) = \cos(2\pi p) + \sin(2\pi p) + \cos(4\pi p) \]
Objects described by basis functions on the sphere.
Challenge: How to get the point coordinates onto the sphere in the first place.
Some spherical harmonics (the radially symmetric ones).

Functions have global support.
Spherical Harmonics

Shape representation with spherical harmonics.
General paradigm of orthogonal fcn. representations

- Find a base object with same topology as target object, w/ arguments $q$, and w/ a uniform tesselation
- Choose a set of appropriate basis functions $b^i(q)$ and sample them on the tesselation vertices to produce orthogonal tuples $b^i$
- Sample the target object with a corresponding tesselation to produce $x$
- Representation is tuple of $a^i$, where the $j^{th}$ entry in $a^i$ is the dot product of the $j^{th}$ coordinate entries in $x$ with $b^i$
Aspects of representation by global orthogonal fcns

- Allows truncation, if basis functions well chosen
- Allows interpolation
- Global, i.e., non-local
- Differs depending on parametrization of target(s)
Wavelets

Wavelets have local support.

Image: Mathworks
Image: Mathworks, Wavelet Toolbox Documentation
Spherical Wavelets

Images: Nain et al.

Functions have local support.
Spherical Wavelets

Shape representation at different resolution levels (left original).

Image: Nain et al.
What parameterization for PCA

• PCA yields Gaussian

• Poor parametrization widens Gaussian

• So choose parametrization that yields the tightest Gaussian (or other probability distribution)
  • But also need ~uniform coverage of surface

Use entropy to measure both tightness of probability distribution and coverage
[Cates et al., Twining et al.]
PCA yields best orthogonal fcn set for training space

- Eigenmodes are D model tuple (from mean), just as with other orthogonal functions (from the zeroth “mode”)
- On average over cases in the space, weighted sum of first k basis functions closest matches cases in a sum of squares sense, all k. That is, best truncation.
Shape representation tuples using orthogonal basis functions

- All previous examples used orthogonal basis functions
  - Wavelets (which have local support) are orthogonal re specific scalings and specific translations
- Coefficients are computed by dot product of points with sampled basis functions
- Tuple of coefficients is representation: can do PCA on those tuples